Applying Fast-Fourier Transform in MAGMAP

This *Applying Fast-Fourier Transform in MAGMAP* guide describes the Fast-Fourier Transform process and demonstrates how to apply this process to your gridded data.

MAGMAP applies filters in the Fourier domain. The MAGMAP FFT filters are typically applied to geophysical potential field data. The filters have a wide range of application: they can remove geologic and cultural noise, perform regional/residual separations, estimate physical properties, such as magnetic susceptibility and rock density and modify the datum at which the data was originally collected, such as up/downward continuation and reduction to the magnetic pole. In addition, an interactive tool provides you with the ability to view the symmetrical filter traces on the same graph as the radially averaged power spectrum and thus tune the filter parameters for the best expected results.

MAGMAP enables you to filter the data in using the following methods:

1. 1-Step FFT process
2. Step-by-Step approach
3. Interactive FFT process

The Step-by-Step approach enables you to control each part of the sequence (prepare grids, apply forward FFT, set filters, and apply inverse FFT), whereas the 1-Step process performs the grid preparation, forward FFT, filtering and inverse FFT steps in one go. The interactive process enables you to visualize the data power spectrum waveform as well as the filter operator. You can interactively refine the filter parameters to yield optimal results.

Applying the Forward Fast-Fourier Transform

After the grid preparation, you apply the forward Fast-Fourier Transformation (FFT).

To Apply the Forward Fast-Fourier Transform

1. From the *MAGMAP* menu, select *Step-by-Step Filtering* and then select **Forward FFT**.

   The *FFT2IN* dialog appears.

2. MAGMAP defaults are intelligently set; the output of the previous step becomes the default of the current step. The *Name of Input Pre-processed Grid File* is already defaulted to the grid outputted from the earlier prepare grid step.

3. Click the **OK** button.

   The system computes the transform and saves it as the name you supplied appended with "*_trn*" in the current folder. You will next apply the Filters to this transform.

⚠ *The transform signal has real and imaginary components; these are saved as interleaved in the output grid. As a result, although the grid format is used for storage, the transform has no meaningful expression visually. The stipulation that the output should require twice the storage space of the input grid is balanced with the fact that the transform of a periodic signal is symmetrical along the Y axis and thus the 3rd & 4th quadrant information becomes redundant.*

The net outcome is that the transform has the same Y dimension as the input grid, but in X, it has to take into account the real and imaginary components of the folding axis and thus its X dimensions will be set to the input X dimension +2.

The FFT Algorithm

MAGMAP uses the Fast Fourier Transform (FFT) algorithm to convert the space domain gridded data to the wave-number or Fourier domain where the filters are applied. By nature the transform of real data is complex and symmetrical. We take advantage of this fact to save the transformed data in a similarly sized grid by discarding the redundant mirror images (folded) part of the transform and storing the complex number as a pair of real and imaginary real 4-byte components

The output transform is saved under the base same name as the input grid, appended with the .TRN extension.

As part of the process, MAGMAP also calculates and saves a radially averaged energy spectrum of the transform.

The size of the transform grid element is 4 bytes; each consecutive pair of elements represents the real and imaginary component of a complex number. The transform is stored in the same way as the input grid, so that each transform vector (row) represents a vector in the storage direction of the input grid (X for kx=1, Y for kx=-1). The following table illustrates the logical storage of the transform:

Vectors	Elements			E(1,2)	E(3,4)	E(5,6)	E(n+1)	E(n+2)
1 / 2e (nyq.)	V(n)	-(1/nv)	r	i r	i r	i *	r	i
	V(n-1)	-(2/nv)	r	i r	i r	i *	r	i
			*	*	*		*	
	V(n/2+2)	-(1/2v - 1/nv)	r	i r	i r	i *	r	i
	V(n/2+1)	1/2v (nyq.)	r	i r	i r	i *	r	i
	V(n/2)	-1/nv	r	i r	i r	i *	r	i
			*	*	*		*	
2 / ne	V(3)	2/nv	r	i r	i r	i *	r	i
1 / ne	V(2)	1/nv	r	i r	i r	i *	r	i
0	V(1)	0	r	i r	i r	i *	r	i

Where:

r, i are real and imaginary components of each transform element

e, v are element and vector separations (cell size)

n is the original grid dimension in cells

The transform element separation (1/ne) and vector separation (1/nv) is in cycles/metre; where ne and nv are the expanded grid dimensions. Although you can set the grid expansion dimensions to be different along the two axis, it is highly recommended to keep them the same and avoid potential anti-aliasing occurring as a result. The Nyquist wavenumber is the largest wavenumber that has been sampled by the grid, and is defined as the inverse of twice the grid cell size (1/2e and 1/2v, which are equal).

Looking at the above table, you can note that each transform vector (row) represents a discrete Fourier row in the direction of the input grid vectors. The Fourier elements within each row start at 0 cycles/metre and extend to the Nyquist wavenumber in 1/ne increments.

As a result, the transform grid has (n/2 + 1) elements per vector, where n is the number of elements per vector in the expanded input grid. The transform is folded at the Nyquist wavenumber in the direction of the grid vectors, so the transform grid has n vectors.

Calculating the Energy Spectrum in MAGMAP

This *Calculating the Energy Spectrum in MAGMAP* guide introduces you to energy spectrum in MAGMAP. The procedures are designed to show you how to calculate and display the radially averaged and 2D energy spectrum.

In addition to producing the Fourier Transform, the Forward FFT option (FFT2IN.GX) also produces a file containing the radially averaged energy spectrum (2D Power Spectrum) in a format similar to the following example:

```
/ 2-D RADIALLY AVERAGED POWER SPECTRUM

/ WAVENUMBER INTERVAL      DWE =  1.428571E-01

/ AVERAGE SPECTRAL DENSITY  LOG(ETOT) =  1.940511E+01

/

/        CYC/KM #_SAMP         LOG_P      3_DEPTH      5_DEPTH

/ ──────────────────────────────────────

  0.000000E+00       1  6.953915E+00  2.017633E-01              *

  1.428571E-01       8  6.591711E+00  3.892244E-01              *

  2.857143E-01      12  5.556448E+00  5.608587E-01  4.363967E-01

  4.285714E-01      16  4.578010E+00  3.591071E-01  3.645031E-01

  5.714285E-01      32  4.267114E+00  1.735434E-01  2.545823E-01

  7.142857E-01      28  3.954922E+00  2.310963E-01  2.454431E-01

  1.000000E+00      40  2.764027E+00  3.829211E-01  2.908998E-01

  .

  .

  .

  9.428572E+00     424 -7.545702E+00  1.084567E-01  1.966618E-02

  9.571428E+00     420 -7.829783E+00 -3.100928E-02  1.257935E-02

  9.714286E+00     416 -7.434367E+00 -3.970939E-02  4.879844E-02

  9.857142E+00     448 -7.687212E+00  2.171140E-01              *

  1.000000E+01     394 -8.213891E+00  2.933830E-01              *

  1.014286E+01     340 -1.024793E+01              *              *

  1.028571E+01     348 -1.104301E+01              *              *

  1.042857E+01     272 -1.107746E+01              *              *
```

The radially averaged energy listed in the third column represents the spectral density (energy) averaged for all grid elements at the wavenumber in the first column. The second column indicates the number of elements that were used to determine the average. The energy is normalized by subtracting the log of the average spectral density.

The 3-DEPTH and 5-DEPTH columns are ensemble magnetic depth estimates based on 3 and 5 point averages of the slope of the energy spectrum (Spector and Grant, 1970). The depth to a statistical ensemble of sources is determined by the following expression:

$$h = -\frac{s}{4\pi}$$

where:

h is depth

s is the slope of the log (energy) spectrum

The above estimates can be used as a rough guide to the depth of magnetic source populations.

The system enables you to create and view a radially averaged spectrum automatically.

Calculating Radially Averaged Power Spectra

To better understand the filter operator, calculate a radially averaged power spectrum in the wavenumber domain and display the filter superimposed with the actual data signal.

To Calculate the Radially Averaged Spectrum

1. From the *MAGMAP* menu, select *Spectrum Calculation and Display* and then select **Radial Average Spectrum**. The *FFTRSPC* dialog appears.
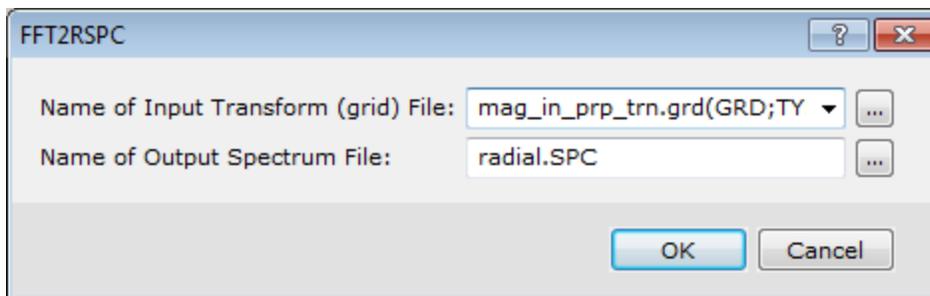
2. Select the *Name of Input Transform (grid) File*.

    ⚠️ *The Input Transform grid file is created when you apply the Forward FFT to your prepared grids and is automatically populated.*

3. Specify the *Name of Output Spectrum File*, the default name is "radial.SPC".
4. Click the **OK** button. The system computes the spectrum.

Displaying Radially Averaged Spectra

You can view the spectra calculated in the previous step in your project. The spectra are automatically formatted to fit on a page when printed. To examine a sample plot, see See "Calculating the Energy Spectrum in MAGMAP" on page 4.

To Display Radially Averaged Spectrum

1. From the *MAGMAP* menu, select *Spectrum Calculation and Display* and then select **Display Spectrum**.

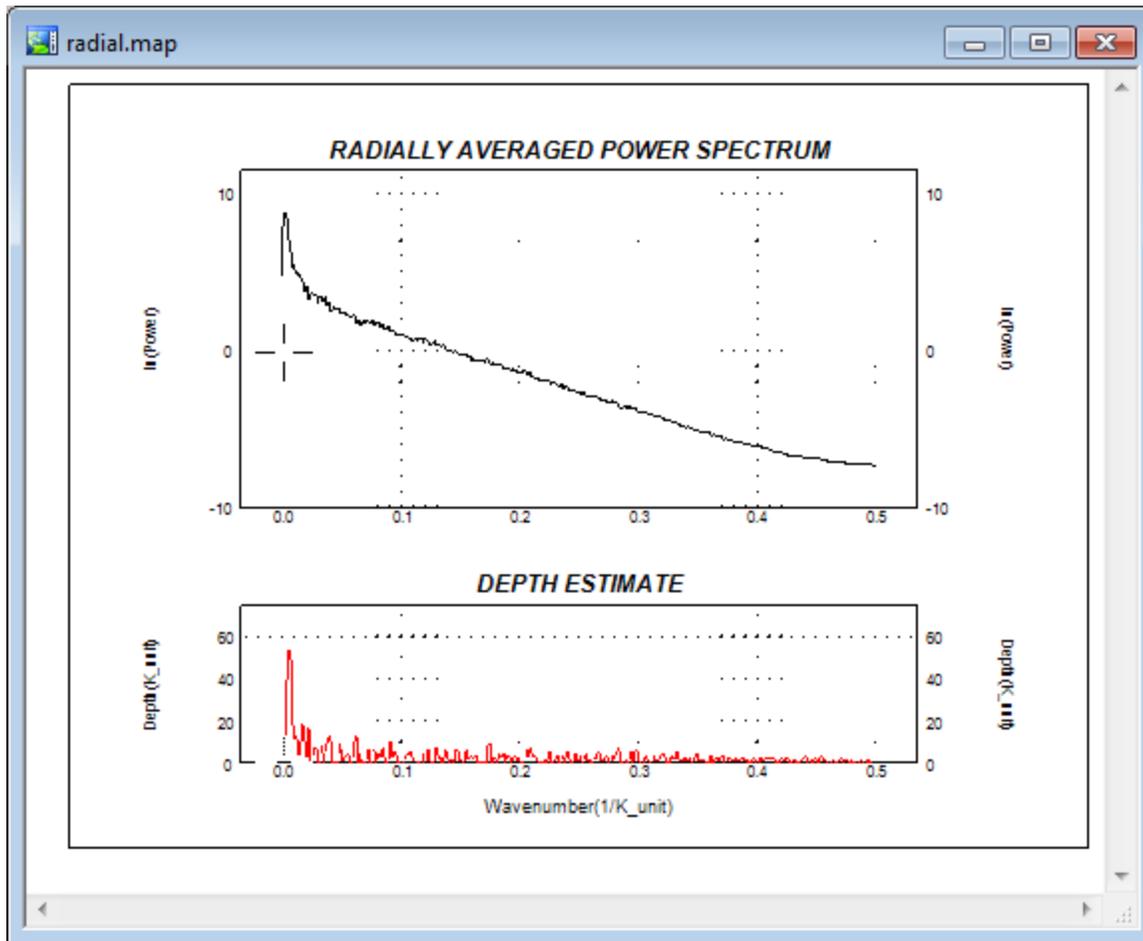   The *Create a spectrum map* dialog is displayed.
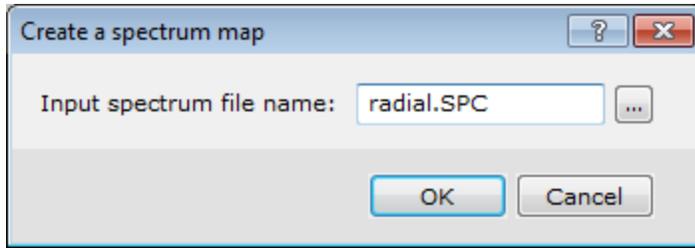
2. Using the *Browse* button locate the *Input spectrum file name*.

   ⚠️ *The Input Spectrum file is created when you calculate the radially averaged spectrum (previous step) and is automatically populated.*

3. Click the **OK** button.

The spectrum map is displayed.

The power spectrum is radially averaged. As a result, the larger the grid, the smoother the power spectrum will be. The file "radial.SPC" is an ASCII file with 5 fields - the wavenumber increment, the number of samples averaged for that wavenumber, the logarithm of the averaged wavenumber, and a 3 and 5 point depth estimate.

The magnetic depth estimate displayed in the lower graph is the average of the slope of the energy spectrum over 5 points. The depth to a statistical ensemble of sources is determined by the expression,
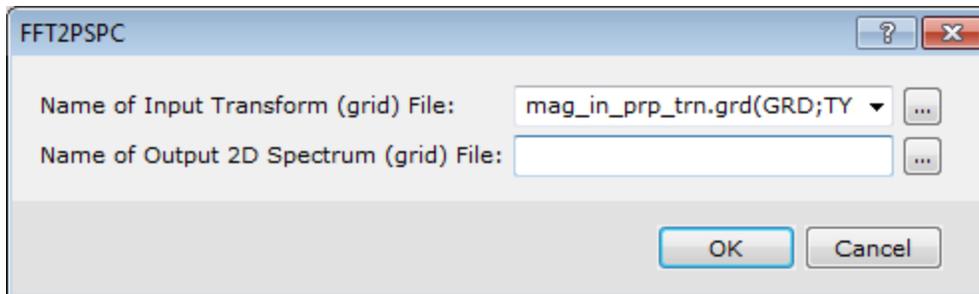
$$h = \frac{-s}{4\pi}$$

Calculating and Displaying 2D Power Spectra

If you are performing a more complex interpretation, you may be interested in computing and displaying the 2D power spectrum for your grid.

To Calculate and Display 2D Power Spectra

1. From the *MAGMAP* menu, select *Spectrum Calculation and Display* and then select **2D Power Spectrum**.
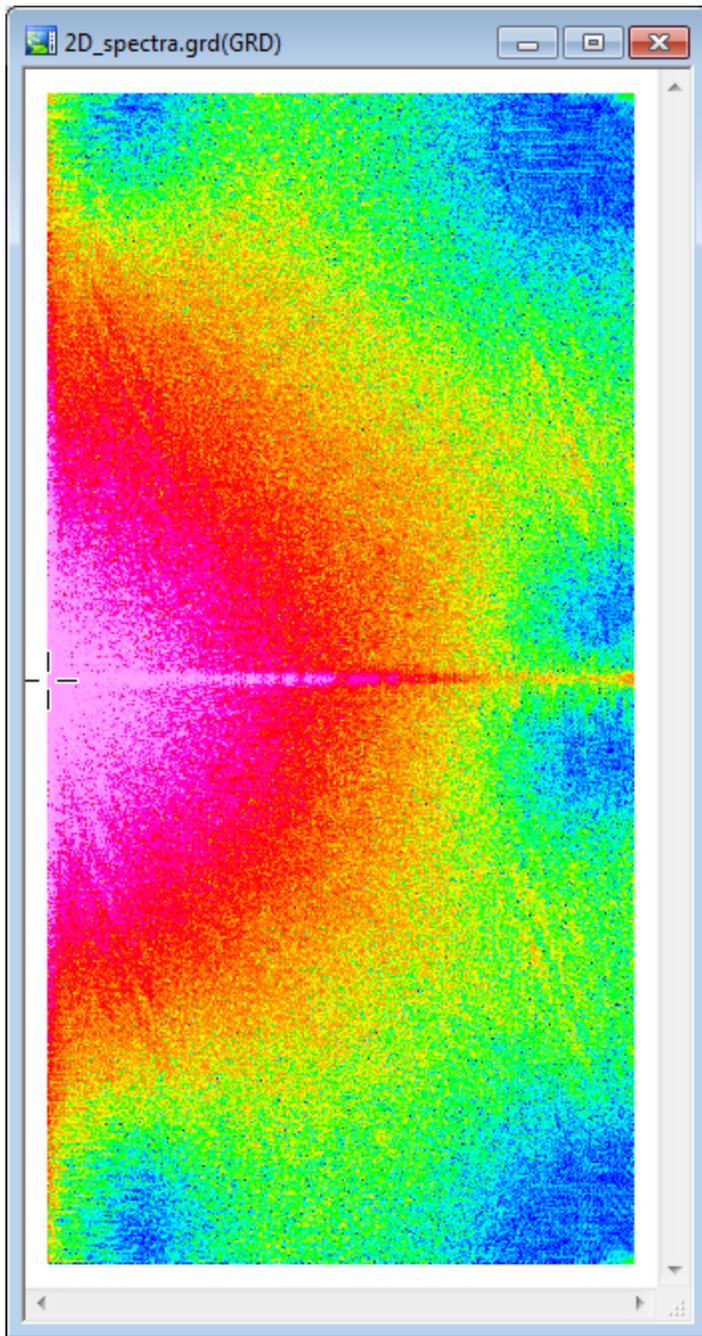
    The *FFTPSPC* dialog is displayed.

2. Use the *Browse* button to select the *Name of Input Transform (grid) File*.

    ⚠️ *The Input Spectrum file is created when you calculate the radially averaged spectrum and is automatically populated.*

3. Specify the *Name of Output 2D Spectrum (grid) File*.

4. Click the **OK** button.

The system computes the spectrum grid and displays it in a temporary map file.

Publication Date: 08/12/2015