

Applying Filters with montaj Geophysics

The following How-To Guide explains how the montaj Geophysics™ system prepares data for 1D-FFT filtering and details the application and mathematics of each of the 1D-FFT filters.

For more information on montaj Geophysics concepts, capabilities and process, see the *getting Started with montaj Geophysics* How-To Guide.

Preparing Data for Filtering

The distance unit used by all filters is the fiducial. This means that if you upward continue the data, for example, you must specify the continuation distance in fiducials, or if you calculate a vertical derivative of total magnetic field data, the units for the result are nT/fiducial.

It is normally more convenient to work with your data in a real distance unit such as metres (or your ground_units). To do this, you must make sure the fiducial numbering of the channel you want to filter is based on distance. If your fiducial numbering is not based on a real distance unit, we recommend that you re-sample your data to a real distance base before applying filters.

To resample the data to a distance base, select the *Advanced Usage|Re-fid to distance* option in the *FFT1D* menu. The corresponding GX (DISTFID) looks for a channel named "_Distance" (containing the along-line distance of each point in the channel). If the "_Distance" channel does not exist, the system creates one based on the "X" and "Y" channels in the database. The GX then resamples the input data channel (using the distance channel as the new fiducial base) and creates a new data channel.

The DISTFID GX also creates a channel with the same name as the original channel, but with extension "_fid" appended. This channel contains the original fiducial values for the new data channel. If you want to return the filtered channels back to the original fiducial base for other processing, you will need the original fiducial values.

To return channels to their original values, select the *Advanced Usage|Re-fid back to fiducial* option under the *FFT1D* menu and specify the filtered channel to convert back to a fiducial base. As the output channel, you may wish to specify the same input channel so that the system simply replaces the channel with the new data. As the fiducial base channel, you must specify the "_fid" channel (for example, "mag_fid") that you created from the original data. Finally, leave the start fiducial blank and specify the original fiducial increment.

Applying Standard Filters

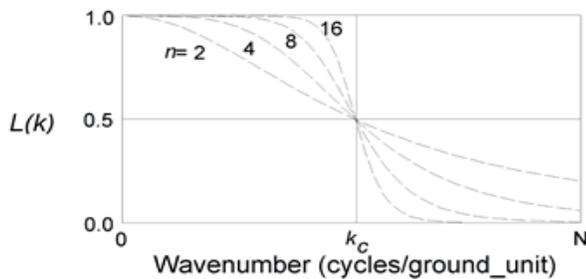
This section describes the use of the individual filter GXs included in the montaj Geophysics system in terms of the application and mathematics of each 1D-FFT filter. Each of these filters starts with a line of data in the space domain and produces a new line of data in the space domain that is the result of applying the filter.

The mathematical expressions use the following terms:

w	Angular wavenumber in radians / ground_unit
k	Wavenumber in cycles / ground_unit ($\omega = 2\pi k$)
N	Nyquist wavenumber [$1/2d$ where d is the original data increment]

Butterworth Filter (FFTBUTTW GX)

The Butterworth filter is excellent for applying straight forward high-pass and low-pass filters to data because you can easily control the degree of filter roll-off while leaving the central wavenumber fixed. If ringing is observed in the filtered data, you can reduce the degree until you are satisfied with the result. The diagram below shows the shape of the filter function for different filter degrees (n). A common, but more complicated alternative is the Cosine filter (FFTCOSN).



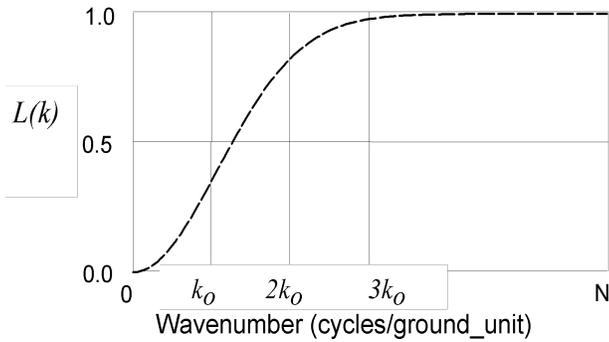
$$L(k) = \frac{1}{1 + \left(\frac{k}{k_c}\right)^n}$$

Parameters:

k_c	Central wavenumber of the filter.
n	Degree of the Butterworth filter function. By default, we assume a degree of 8.
$0/1$	Residual/Regional flag to specify if a residual high pass or a regional low pass is required. By default, the system applies a regional filter.

Gaussian Regional/Residual Filter (FFTGAUSS GX)

The Gaussian filter is a filter often used for low-pass or high-pass applications.



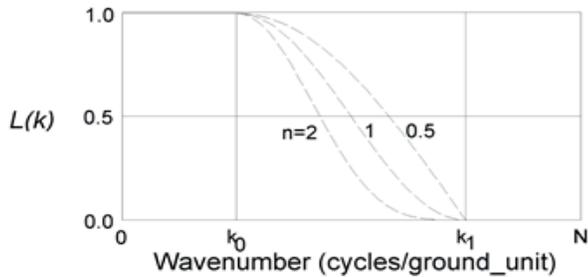
$$L(k) = 1 - e^{-\left(\frac{k^2}{2k_0^2}\right)}$$

Parameters:

k_0	Standard deviation of the Gaussian function in cycles/ground_unit (similar to a cutoff except that the function amplitude at this point is only 0.39).
$0/1$	Specify 0 or 1 for residual or regional, respectively.

Cosine Roll-off Filter (FFTCOSN GX)

Because this filter has a smooth shape and does not alter the energy spectrum below the start of roll-off (or after the end of roll-off in high-pass mode), it is commonly used for simple high-pass or low-pass operations. To reduce ringing, you can increase the separation between k_1 and k_0 .



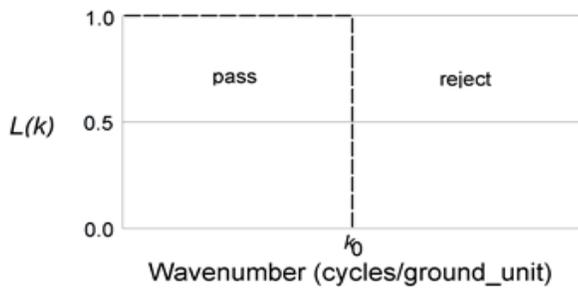
$L(k) = 1$	for $k < k_0$
$L(k) = \cos^n \left[\frac{\pi}{2} \left(\frac{k - k_0}{k_1 - k_0} \right) \right]$	for $k_0 \leq k \leq k_1$
$L(k) = 0$	for $k > k_1$

Parameters:

k_0	Low wavenumber starting point of the filter. (Cut-off wavenumber for high-pass or start of roll off for low-pass.)
k_1	High wavenumber end point of the filter. (Start of roll off for high-pass or cutoff wavenumber for low-pass.)
n	Degree of the cosine function. The default is a degree of 2 for a cosine squared roll-off.
$O/1$	0 for residual (high-pass) filter; 1 for regional (low-pass) filter. The default is a low-pass filter.

Lowpass Filter (FFTLOWP GX)

As with the band-pass filter, you should use this filter selectively because it can introduce Gibb's Phenomena (ringing) into the filtered output data.



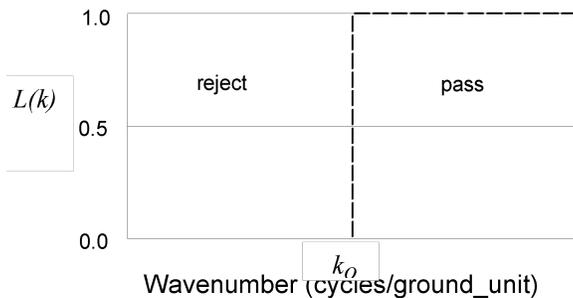
$L(k) = 1$	for $k < k_0$
$L(k) = 0$	for $k > k_0$

Parameter:

k_0	Cutoff wavenumber in cycles/ground_unit. All wavenumbers above this value are removed.
-------	----------------------------------------------------------------------------------------

Highpass Filter (FFTHIGHP GX)

As with the band-pass filter, you should use this filter selectively because it can introduce Gibb's Phenomena (ringing) into the filtered output data.



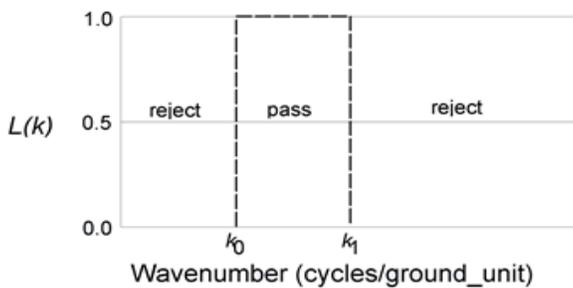
$L(k) = 0$	for $k < k_0$
$L(k) = 1$	for $k \geq k_0$

Parameter:

k_0	Cutoff wavenumber in cycles/ground_unit. Removes all wavenumbers below this value.
-------	------------------------------------------------------------------------------------

Bandpass Filter (FFTBANDP GX)

You can use the Bandpass filter to pass or reject a range of wavenumbers from the data. However, applying such a simple cutoff filter to an energy spectrum almost invariably introduces a significant amount of ringing into the filtered output data. We recommend that you use a smoother filter such as the Butterworth filter (FFTBUTTW).



$L(k) = 0$	for $k < k_0$
$L(k) = 1$	for $k_0 \leq k \leq k_1$
$L(k) = 0$	for $k > k_1$

Parameters:

k_0	Low wavenumber cutoff in cycles/ground_unit.
k_1	High wavenumber cutoff in cycles/ground_unit.
0/1	If 1, pass the defined band. Otherwise, reject the defined band. The default is to pass the band.

Vertical Derivative (FFTVDRV GX)

The vertical derivative is commonly applied to total magnetic field data to enhance the most shallow geologic sources and suppress deeper sources in the data. The order of differentiation defines the high-wavenumber component to enhance; increasing the order will enhance higher-wavenumber components of the spectrum. As with other filters that enhance the high-wavenumber components of the spectrum, you must often also apply low-pass filters to remove high-wavenumber noise. Note that you can apply fractional orders of differentiation (e.g. 0.5) to reduce high-wavenumber noise.

$$L(\omega) = \omega^n$$

Parameter:

n	Order of differentiation.
-----	---------------------------

Vertical Integration (FFTVINT)

This filter calculates the vertical integral of the input transform. This is the inverse of the vertical derivative. The zero wavenumber is set to 0.

$$L(\omega) = \omega - 1$$

Horizontal Derivative (FFTHZDRV GX)

The horizontal derivative enhances high frequency horizontal variations in potential field data. Such variations can be caused by faults and/or boundaries between different geological units; as such horizontal derivative data can be useful for detecting/interpreting these features.

$$L(\omega) = (\omega i)^n$$

Parameter:

n	Order of differentiation.
-----	---------------------------

Horizontal Integration (FFTHZINT GX)

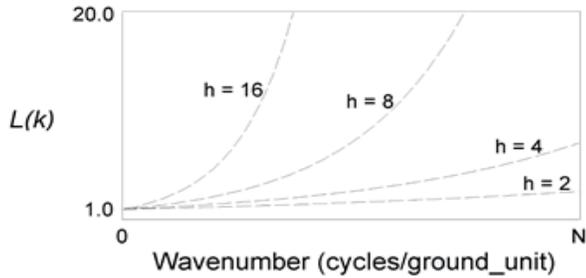
The Gradient Inversion (Horizontal Integration) filter calculates the horizontal integral of the input transform. This is the inverse of the horizontal derivative. The zero wavenumber is set to 0.

$$L(\omega) = i\omega - 1$$

Downward Continuation (FFTCONT GX)

Downward continuation enhances the response from sources at a shallow depth (by effectively bringing the plane of measurement closer to the sources). Note, however, that it is not possible to continue through a potential field source. If the data contains short wavelength noise, this noise can appear as very shallow sources in the continuation. Such noise should be removed before attempting to downward continue the data. A Butterworth low pass filter set to between 1 and 1.5 times the depth can be very effective to remove noise before continuation.

You should make a plot of the energy spectrum to determine the wavenumber at which sources (noise) appears to be more shallow than the depth of continuation. The energy spectrum is also a good guide for determining the depth to which you can continue data downward.



$$L(\omega) = e^{h\omega}$$

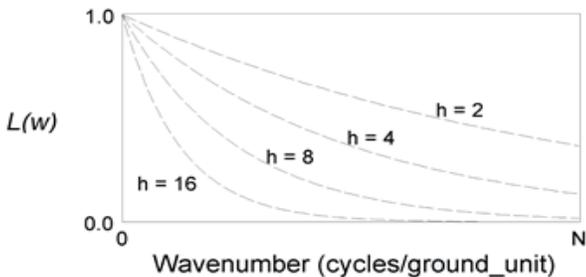
Parameter:

<i>h</i>	Distance in ground_units, to continue down relative to the plane of observation.
----------	----------------------------------------------------------------------------------

Upward Continuation (FFTCONT GX)

Upward continuation is considered a clean filter because it produces almost no side effects that may require the application of other filters or processes to correct. Because of this, it is often used to remove or minimize the effects of shallow sources and noise in grids.

Also, you can interpret upward continued data numerically and with modeling programs. This is not the case for the output from many other geophysical filters.



$$L(\omega) = e^{-h\omega}$$

Parameter:

<i>h</i>	Distance in ground_units, to continue up relative to the plane of observation.
----------	--------------------------------------------------------------------------------

Magnetic Pole Reduction (FFTRPOLE GX)

Reduction to the Pole (RTP) is a filter that recalculates total magnetic intensity data as if the inducing magnetic field had a 90 degree inclination. For example a magnetic anomaly occurring at a low magnetic latitude may be dipolar and offset from its causative body; RTP transforms this into a monopolar anomaly that is centred over the causative body, thus simplifying interpretation.

RTP assumes that all magnetisation is parallel to the Earth's magnetic field; therefore it should be used with caution where remanent magnetisation is known or suspected to comprise a significant component of the total field.

The reduction to the pole is:

$$L(\theta) = \frac{1}{(\sin(I_a) + i\cos(I) \cdot \cos(D - \theta))^2}$$

where:

I	Geomagnetic inclination
I_a	Inclination for amplitude correction (never less than 1)
D	Geomagnetic declination

Parameter:

I_a	Inclination to use for the amplitude correction. Default is 20. $I_a = 20$, if $I > 0$; $I_a = (-20)$, if $I < 0$. If $ I_a $ is specified to be less than $ I $, it is set to I .
-------	-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

Reduction to the pole has an amplitude component (the $\sin(I)$ term) and a phase component (the $i\cos(I)\cos(D - \theta)$ term). When reducing to the pole from equatorial latitudes, North-South features can be enhanced due to the strong amplitude correction (the $\sin(I)$ term) that is applied when $D - \theta$ is $\pi/2$ (i.e. a magnetic east-west wavenumber). By specifying a higher latitude for the amplitude correction alone, this problem can be reduced or eliminated at the expense of under-correcting the amplitudes of North- South features. If this does not fix or reduce the problem, it may be better to use the Analytic Signal filter instead.

An amplitude inclination of 90 causes only the phase component to be applied to the data (no amplitude correction), and a value of zero causes phase and amplitude corrections to be applied over the entire range.

Apparent Susceptibility (FFTSUSC GX)

The susceptibility filter calculates the apparent magnetic susceptibility of the magnetic sources using the following assumptions:

- The IGRF has been removed from the data.
- There is no remanent magnetization.
- All magnetic response is caused by a collection of vertical prisms of infinite depth and strike extent.

A susceptibility filter is, in fact, a compound filter that performs a reduction to the pole, downward continuation to the source depth, correction for the geometric effect of a vertical prism, and division by the total magnetic field to yield susceptibility.

$$L(k, \theta) = \frac{1}{2\pi F \cdot H(\omega) \cdot \Gamma(\theta) \cdot K(k)}$$

$$H(\omega) = e^{-h\omega}$$

$$\Gamma(\theta) = (\sin(I_a) + i \cos(I) \cdot \cos(D - \theta))^2$$

$$K(\omega) = \left(\frac{\sin(a\omega)}{a\omega} \right)$$

where:

$H(\omega)$	Downward continuation to h
$\Gamma(\theta)$	Reduction to the pole
$K(\omega)$	Geometric factor of a vertical prism -- ($2a \times \omega \times \omega$ in dimension)

Parameters:

h	Depth in ground_units, relative to the observation level at which to calculate the susceptibility.
I_a	Pole reduction amplitude inclination. Inclination to which to use the phase component only in the reduction to the pole. The default is 20. If $ I_a $ is specified to be less than $ I $, it is set to I .
I	Geomagnetic inclination
D	Geomagnetic declination
F	Total geomagnetic field strength

Apparent Density (FFTDENS GX)

This filter calculates the apparent density of the ground that would give rise to the observed gravity profile. The density assumes that the gravity profile is due to a set of rectangular prisms with a top at the level of observation of the gravity profile, a bottom at depth t , and infinite strike length.

You may wish to downward continue the profile to be close to the tops of the assumed geologic model of interest before calculating the apparent density.

You must supply the thickness of the earth model and the background density.

$$L(\omega) = \frac{\omega}{2\pi G(1 - e^{-t\omega})}$$

Parameters:

t	Thickness, in ground_units, of the earth model.
d	Background density in g/cm3
G	Gravitational constant.

Multiple Filters (FFT1MULT GX)

The Multiple Filters option enables you to apply multiple 1D Fast-Fourier Transform filters to a channel.

The Multiple Filters option allows multiple filters to be applied to a channel with a single process, in which the data is transformed into the wavenumber domain, multiple specified filters are applied (simple multiplications), and the data is transformed back into the space domain.

 *Note that in the 1DFFT specific single filter GX (fftvdrv GX, fftbuttw GX, etc...) the application of each filter requires its own FT-filter-IFT process. This tends to add numerical noise to the data as the number of filters is increased. Using this GX for multiple filters will avoid this problem.*

In this GX, along with the input and output channel names, the user can select up to six filters to apply. A separate dialog is displayed for each filter type specified, in which the user is prompted to enter values for the parameters relevant to that filter.

For a non interactive process application, the script parameters for each filter type specified can be referred to the relevant 1DFFT GXs. Information on each filter type is available by clicking the [Help] button on the individual filter parameter dialogs.

Analytic Signal (FFTAS GX)

The Analytic Signal option calculates the analytic signal of a channel. The analytic signal can be useful for locating the edges of remanently magnetized bodies and for centring anomalies over their causative bodies in areas of low magnetic latitude (Macleod et al., 1994).

The analytic signal (as) of a profile is defined as:

$$as = \sqrt{dz \cdot dz + dx \cdot dx}$$

where:

dz	is the vertical derivative
dx	is the horizontal derivative

The vertical derivative is calculated using the FFT process described in a separate guide. The horizontal derivative is calculated by applying a space domain convolution filter. The analytic signal is then evaluated from these two sets of data.

Hilbert Transform (HILBERT GX)

The Hilbert Transform option does the Hilbert Transform by the means of FFT based on the following known relation:

$$f[H[f(x)]] = -i \cdot \text{Sgn}(\omega)F[f(x)]$$

Where:

$F[f(x)]$	is the Fourier transform of $f(x)$
$H[f(x)]$	is the Hilbert transform of $f(x)$
$\text{Sgn}(\omega) = \frac{\omega}{ \omega } = +1$	$= +1$ for $\omega > 0$; $= 0$ for $\omega = 0$; $= -1$ for $\omega < 0$

1. Firstly, the GX does forward FFT transform of the input channel. Three output channels are created (this is only for the none-array input channel case). They will have the same name as the input channel but extension "_r" and "_i" for real and imaginary components of the transform, and "_w" for the wavenumber in radians/fiducial.

 *Note that the trend has been removed before FFT.*

 *Note that the output values are the real and imaginary components of the positive frequencies of the transform. Since we are dealing with the real-valued space domain problem, the negative part of the spectrum is simply the conjugate of the corresponding positive part, i.e., $h(-f) = [h(f)]^*$, and is not included in the output.*

2. The fiducial number will be in cycles/fiducial. The wavenumber channel will be in radians/fiducial.
3. Then, the one-dimensional Hilbert transform operator $-i \text{sgn}(w)$ is applied to the FFT transformed data.
4. Finally, the GX does the inverse FFT transform to obtain the Hilbert transform results into the output channel.

For the real data practice, it is suggested to remove trend line based on all data points (the default) before FFT process to prevent the discontinuity from the data two edges. The removed trend will be replaced back in the same manner as it removed after FFT.

For the real data practice, it is also suggested to expand the data 10% before FFT process to prevent the discontinuity from the data two ends. The GX will first extend data by the user required % points, then further extend to the number of the power of 2 for the FFT process. For instance, if the original data contains 60 points, then the values will be padded with 10%, or 6 points at the end, giving 66 points. This will then be extended to the next power of 2, or 128 points, to do the FFT.

The extended area will be interpolated by *Maximum Entropy Prediction (MEP)* method. MEP samples the original data points to determine its spectral content. It then predicts a data function that will have the same spectral signature as the original data. As a result, the predicted data will not significantly alter the energy spectrum that would result from the original data alone.

However, for particular synthetic data test the set of "remove mean value" trend line removal option and 0% expansion may obtain accurate results.

Reference Paper: "Toward a three-dimensional automatic interpretation of potential field data via generalized Hilbert transforms: Fundamental relations", Misac N. Nabighian, Geophysics, Vol. 49, No. 6, pp. 780-786, 1984.

Power Spectrum (FFTPSPEC GX)

The Power Spectrum option will calculate the log power spectrum of a channel.

 *Note that, the power spectrum will be a log base 10 of the power.*

The fiducial increment will be cycles per original fiducial. The profile is extended to a power of 2 times the length and the wavenumber increment will be

$$1/(length * fid_increment)$$

Applying Custom Filters

The montaj Geophysics system also enables you to work with the transform directly so that you can apply your own custom filters.

The *Advanced Usage* menu provides users with step-by-step 1D-FFT data processing sub-menus. These menu items are designed for advanced users who want to apply custom filters or math expressions to their data.

Set Expansion Method...	Set expansion method FFTSETEXTMD GX
Preprocess...	Preprocessing previewer FFTPREP.GX
Re-Fid to Distance...	Re-fid channel to distance base DISTFID.GX
FFT Space -> Fourier...	Transform from space to wavenumber domain. FFTIN.GX
FFT Fourier -> Space...	Transform from wavenumber to space domain. FFTOUT.GX
Re-Fid Back to Fiducial...	Re-fid channel to fiducial base REFID.GX

For detailed information on these or any of the 1D-FFT dialogs, click the **Help** button.

The FFTIN GX (*FFT1D | Advanced Usage | FFT space --> Fourier* menu item) transforms a channel to the Fourier domain and creates a real and imaginary channel and a wavenumber channel. You can then apply filters with the real, imaginary and wavenumber channels to create new real and imaginary channels. The FFTOUT GX (*FFT1D | Advanced Usage | FFT Fourier --> Space...* menu item) transforms the filtered real and imaginary channels back to the space domain and masks the result against the original channel.

The channels created by FFTIN have the same name as the original input data channel with the suffixes “_r”, “_i” and “_w” added. These are the real, imaginary and wavenumber channels respectively. The wavenumber channel is in units of radians/fiducial (2p cycles/fiducial). The fiducial numbering of the Fourier domain channels is in cycles/fiducial.

 *Note that the FFTIN process does not remove a trend from the data. If your data contains an undesired trend, you must remove it before running FFTIN. It is also your responsibility to replace a trend in the data after processing.*

The process of applying your own filter is best illustrated through an example. Say we have an existing channel named “Mag” which is already sampled on a distance base (the fiducials are ground_units). We would like to calculate the first vertical derivative using the expression:

$$L(\omega) = \omega^n$$

where n is the order of differentiation (in this case, $n=1$). To apply this expression we must multiply the Fourier transform by the wavenumber:

1. On the *FFT1D | Advanced Usagemenu*, select **FT space -->Fourier** and select the channel to transform. This creates channels appending “_r”, “_i” and “_w” to the selected channel <name>.
2. Select the entire “<name>_r” channel in the database (click three times on the “<name>_r” channel header).
3. Press the “=” (equal sign) to enter an expression, and enter:
“<name>_r*<name>_w”
The “<name>_r” channel is replaced by the filtered real data.
4. Select the entire database for the “<name>_i” channel. Press “=” to enter an expression, and enter: “<name>_i*<name>_w”. The “<name>_i” channel is replaced by the filtered imaginary data.
5. Select **FFT Fourier --> Space...** and choose the “<name>_r” and “<name>_i” as the input channels, “<name>” as the original reference channel, and “dz” as the output filtered channel. The “dz” channel will contain the vertical derivative in nT/ground_units.

How-To Guide Publication Date: 16/01/2013

Copyright 2013 Geosoft Inc. All rights reserved.